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John F. Van Derlofske

Lighting Research Center  
Rensselaer Polytechnic Institute, Troy, NY 12180  
[www.lrc.rpi.edu](http://www.lrc.rpi.edu)

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# Computer modeling of LED light pipe systems for uniform display illumination

John F. Van Derlofske\*

Lighting Research Center, Rensselaer Polytechnic Institute, Troy, NY 12180

## ABSTRACT

Computer modeling of distributive light pipe systems using light emitting diode (LED) sources to produce uniform illumination for liquid crystal displays (LCDs) is presented. Due to their small size, lower power consumption, and lower heat generation, LEDs are a natural source choice for display illumination. However, to be useful in display applications, LEDs must be made to produce uniform illumination over the display area. The conversion of an LED's output flux distribution to one that is uniform over a given area can be accomplished with plastic, injection-molded light pipes. Illustrative examples of LED light pipe display systems are presented. These systems compare output coupling surface geometry for two LED input coupling scenarios; direct and indirect input coupling. Computer modeling via commercial software packages is used to optimize and analyze system designs. It is critical in these simulations to have accurate source models. Therefore close attention is paid to the LED source model. The final simulation results are presented and uniformity and total light output is compared. The implications of these results for display applications are discussed.

**Keywords:** LEDs, backlighting, nonimaging optics, ray tracing, light pipes, illumination.

## 1. INTRODUCTION

In today's complex and connected society the display of information has become crucial to everyday life. Whether it is driving in your car or working remotely on a laptop computer or even operating kitchen appliances, information needs to be efficiently conveyed. For safety and esthetic reasons displays must be easily readable and non-distracting. This implies that regardless of the application displays must be adequately bright, provide sufficient contrast, and be uniformly illuminated.

Although new display technologies are rapidly advancing, liquid crystal displays are still predominantly used for displaying information. Applications range from relatively large area lap top computer screens to medium area entertainment playback device screens to small area cell phone displays. In and of themselves LCDs produce no light, but rather are transmissive viewing devices. Therefore, it is critical to provide backlighting solutions that are compact, optically and energy efficient, inexpensive, and lightweight.

Although there are many lighting technologies and configurations for illuminating LCDs, such as cold cathode fluorescent lamp (CCFL) edge lit systems and direct electroluminescent (EL) illumination, this paper considers LED systems. As a technology LEDs can offer the advantages needed for display applications. They are compact, lightweight sources that use little energy, generate little heat, are robust, and have long life. LEDs produce sufficient light in a variety of colors, including white. The light from LEDs can also be mixed to produce most desired colors. The challenge in using LEDs for backlighting applications is to develop efficient inexpensive systems that take the light from these fairly directional sources and distribute it uniformly over a large area. Distributive light pipe systems offer a solution for this application. Light pipes can be manufactured through injection molding, which is a relatively fast, inexpensive, easily manufacturable, accurate method of fabrication that is suitable for display applications.[1]

When developing LED illumination systems for LCD backlighting the design goal is to provide sufficient light to meet brightness requirements and provide uniform illumination over the entire LCD panel. In order to reduce development and manufacturing costs, the number of system elements, such as LEDs and diffusing layers, must be minimized and the simplest geometries should be used. The total amount of light needed will dictate the minimum number of LEDs in the system. However, the light from these LEDs must be used as efficiently as possible to eliminate the use of additional LEDs and minimize the number of diffusing elements. It is much more preferable to design the light pipe to provide uniform illumination over the entire area of the display than to add additional LEDs and diffusing elements to achieve the same goals.

\* Correspondence: [vandej3@rpi.edu](mailto:vandej3@rpi.edu); [www.lrc.rpi.edu](http://www.lrc.rpi.edu); (518) 687-7100

Computer modeling provides a very useful tool for developing and analyzing light pipe illumination systems.[2][3] Through computer modeling this paper examines the design of efficient LED light pipe systems to provide uniform illumination for LCD backlighting. Specifically, the shape of the bottom output-coupling surface is examined comparing flat, linear, and curved aspheric geometries for total output and uniformity. Accurate system models, including LED sources and light pipe geometries, are developed in a commercial optical design software package. Using ray trace analysis the systems are analyzed and optimized for output uniformity. Modeling results, such as total output flux and output illuminance distributions, are presented and compared. Conclusions are made on the efficiency of surface geometries and then generalized to other cases.

This analysis is performed for two input coupling geometries: direct and indirect. Two input geometries are examined to illustrate that optical design solutions can be achieved for most input distributions. It is shown that aspheric surfaces result in optimal design providing uniform illumination. These results are not unexpected. Other research has shown that efficient development of aspheric surfaces for plastic light pipe systems is possible and that these surfaces can provide superior optical performance. [3][4][5][6]

Several assumptions are made to limit the scope of this discussion. The displays considered here are of medium to small area and have area aspect ratios that are longer than they are wide. Large and/or square display areas require different design strategies. Also, the surface treatments for output coupling considered here scatter homogeneously. Scattering optics can be designed to scatter preferentially in order to increase uniformity. This is a separate design strategy and is not considered in the scope of this paper.

## **2. ILLUMINATION SYSTEM GEOMETRY**

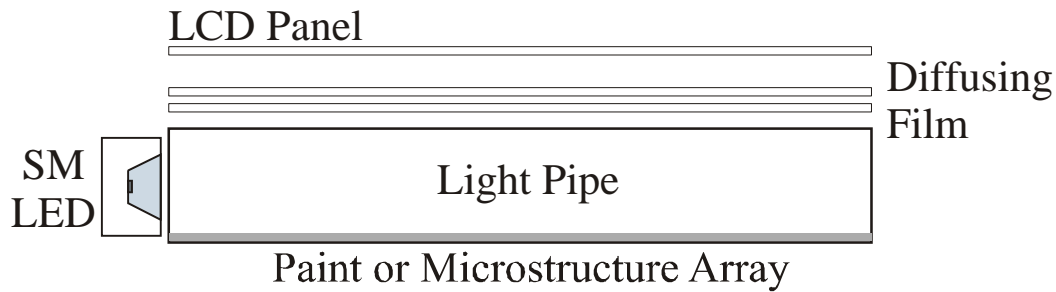
For any light pipe illumination system the basic optical principals are the same. The light has to couple from the source into the light pipe. Here we consider LEDs as the light source. The input coupled light then travels in the light pipe through the process of total internal reflection (TIR). The propagating light is eventually output coupled to perform the illumination task. Here we consider surface treatment, either the application of paint or a microstructure array, as the means of output coupling. Since these three basic stages are universal, most LED light pipe systems used for backlighting have common geometrical characteristics.

In this section we describe the basic system geometries used for this analysis. The basic geometry consists of surface mounted LED sources and a plastic injection molded light pipe. The light pipe uses its bottom surface to output couple the light through the surface shape and a layer of diffuse white paint. The specific LED sources and light pipe systems used here are modeled after a particular real world application, an automotive PRNDLE LCD display. However, they also are characteristic of these types of systems and illustrative of the problem in general. The geometries are broken down into two categories for discussion; direct input coupling and indirect input coupling.

### **2.1 Direct Input Coupling Geometry**

Direct input coupling systems couple the light directly from the LED into the light pipe with no intermediate steps. This is typically accomplished by placing the emitting surface of the LED as close to the input surface of the light pipe as allowed for by manufacturing tolerances. This is done to increase coupling efficiency and maximize light into the light pipe.

The basic LED illumination system geometry is shown in Figure 1. This figure shows a “slab” light pipe or a flat bottom surface geometry. A right-angle surface mounted LED is used as the light source. A plastic light pipe acts to collect the light from the LED, transport the light down its length, and output couple the light in the desired direction. The output coupled light passes through one or more layers of diffusing film to further scatter the output illuminance distribution and increase uniformity. Finally, the light impinges on an LCD panel to provide backlighting illumination.



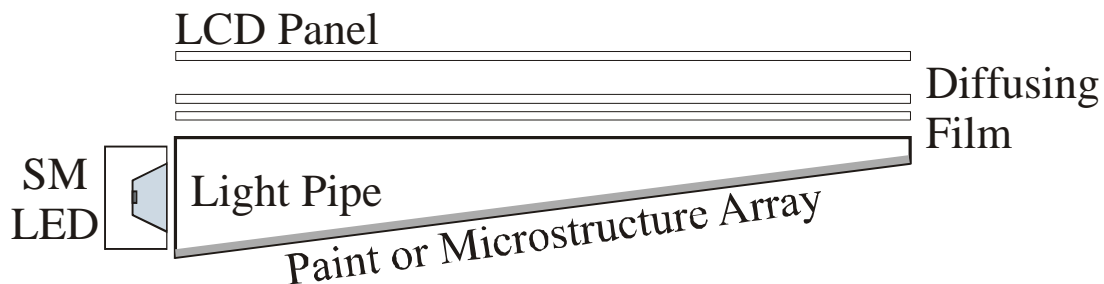
**Figure 1.** Basic LED illumination system geometry for LCD backlighting. This is the slab light pipe or flat bottomed surface geometry.

The right-angle LED is functionally similar to standard surface mounted LEDs except that the packaging is such that the emitting portion of the LED is at 90° to the printed circuit (PC) board. This allows for straightforward input coupling into the light pipe. The number of LEDs used in the system depends on the width of the light pipe (into the page). The systems have illumination area that is longer than it is wide. It is in the long dimension that uniformity becomes an issue. In the shorter width dimension uniformity can be achieved by using multiple LED sources.

Typically, polycarbonate or acrylic plastic materials are used for the light pipe. The dimensions of the light pipe depend on the area to be illuminated. This analysis considers light pipe systems with dimensions of 25 mm – 100 mm and a width of 5 mm – 40 mm. The height of the light pipes ranges from 3 mm – 6 mm.

White diffuse paint or a microstructure array applied to the bottom surface of the light pipe output couples the light by scattering. The paint is applied to the bottom surface in a pad printing operation. Microstructure arrays consist of prisms, lenses, dots, grooves, or other optical geometries that have dimensions on the order of microns. These arrays can be directly applied to the bottom surface of the light pipe, by inmolding or laser etching, or can be applied onto the surface as a film.

Figure 2 and Figure 3 show the other light pipe system geometries examined in this paper. All parameters are identical to the slab light pipe geometry in each case except the bottom surface shape. Figure 2 shows a “wedge” or linear angled bottom surface geometry light pipe system. In this case the bottom surface remains flat but is tilted at some angle with respect to the top surface. Parameters that are optimized for a wedge light pipe are the surface tilt angle and position of the tilted surface. The figure shows the bottom surface tilted right at the input surface. However, the tilt could start at any point along the length of the bottom surface. Figure 3 shows an aspheric bottom surface geometry light pipe system. In this case the bottom surface is tilted and curved in an aspheric shape. Parameters that are optimized for an aspheric light pipe are the surface tilt angle, position of the tilted surface, and surface curvature. This offers one more degree of design freedom for optimization over the wedge light pipe case. Note that for the aspheric case the curvature is only in one dimension. The surface remains flat in the dimension in and out of the page. This design assumption was made to simplify the geometry for discussion. If the display area increases in width aspheric surface shapes can be designed and optimized in both dimensions.



**Figure 2.** Wedge light pipe or angled linear surface geometry.

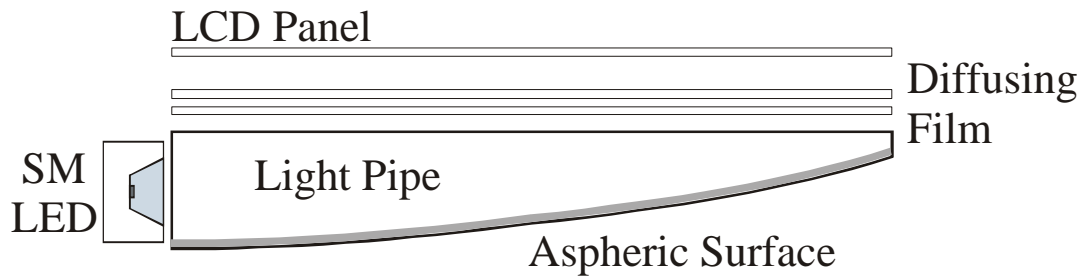


Figure 3. Aspheric light pipe geometry.

## 2.2 Indirect Input Coupling Geometry

Indirect input coupling systems couple the light from the LED into the light pipe with an intermediate step. Figure 4 shows an example of indirect input coupling. Here a diffuse reflector is used to collect light from the LED source and direct it into the light pipe. In this case a standard surface mounted LED is used and the diffuse reflector folds the light 90°. The geometry of the rest of the system is the same as the direct coupling case. The diffuse reflector may be used for two reasons. One is to scatter the light before it enters the light pipe and increase uniformity. The other is to mix the light from two different colored LEDs to produce the desired display color. A specular reflector can also be used to bend the light 90° for indirect input coupling. However, the uniformity and mixing benefits would not be seen.

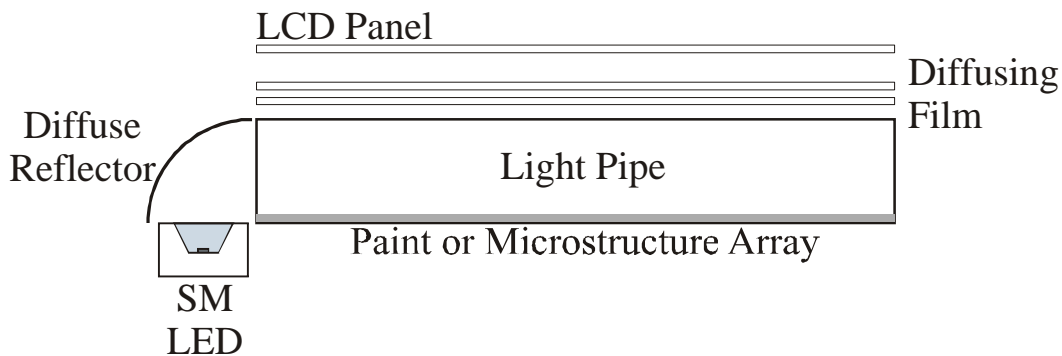


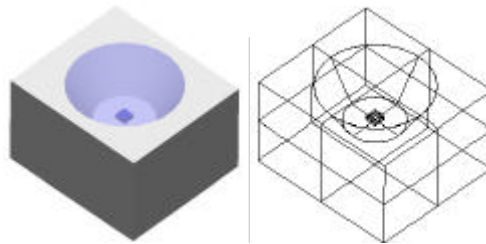
Figure 4. Indirect light pipe system geometry.

## 3. LED SOURCE MODELING

When using computer modeling as a design and analysis tool it is critical to start with an accurate representation of the source. In order to determine optimal methods of converting LED output distributions for LCD display backlighting it is important to start with an LED model that produces an output distribution as close to an actual source as possible. Otherwise the design will have no real application validity.

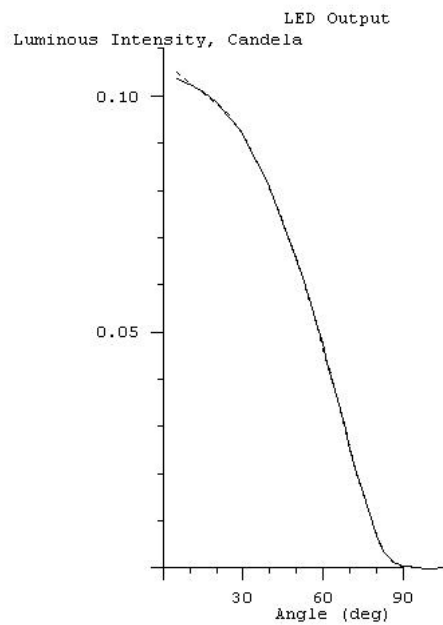
There are two strategies for computer modeling LEDs. One method is to not use a physical model of the LED at all, but rather approximate a uniform emitter or use a predefined ray set to match the measured or specified light output. This approach is good for first order designs where the basic system geometry is determined. Since light rays do not have to be traced through the LED and encounter fewer surfaces this approach is computationally faster. However, this approach does not ensure accurate analysis and can result in erroneous conclusions. The other method for modeling LEDs is to create an accurate physical representation, closely simulating the actual geometry and optical properties. Although this approach is computationally slower it allows for more accurate results. A physical model of the LED allows interaction of scattered light that travels back into the system. It also ensures that both the near and far field intensity distributions are accurate. This is the approach used here to model the surface mounted LEDs.

Figure 5 shows a solid and wireframe representation of the surface mounted LED model. This model represents a commercially available surface mounted LED commonly used for LCD backlighting. It has a total light output of 0.3 lm and a viewing angle of 120°. The relatively wide output distribution is ideal for this application. The housing outer dimensions are 3.4 mm by 3.0 mm by 2.1 mm. The housing material is assumed to be a white diffusely scattering plastic with a small amount of absorption, 10%. The optical characteristics of the housing material are important since the housing also acts as a reflector for the emitting die. The reflector geometry is a tapered cylinder that starts with a 2.3 mm diameter and tapers to a diameter of approximately 1.2 mm at the bottom where the die sits. The reflector cavity is filled with optical epoxy or plastic encapsulant. The epoxy is assumed to have an index of refraction  $n_{\text{epoxy}}=1.5$ . Care is taken in the optical design to ensure that the reflector surface and the epoxy are in optical contact and that no air gap is present. The die is positioned at the center of the reflector cavity and is immersed in the epoxy material. This assures that the light emitted from the die propagates from inside the encapsulant. The die has the correct physical geometry and emission properties to generate the specified output distribution for this LED type.



**Figure 5.** Solid and wireframe representation of the LED source model.

Figure 6 shows the intensity distribution of the LED model. One million rays with a total amount of flux of 0.3 lm were traced from the LED die and collected on a far field intensity receiver. The results are shown graphed as intensity in candela versus viewing angle in degrees. The error in this simulation due to finite sampling was less than 3%. The graph shows that the LED has a peak intensity of ~0.11 cd and a viewing half angle of 60°. This intensity distribution was compared to that specified for this LED and had a correlation factor of  $r^2 = 0.98$ . From these results it is evident that the physical and optical parameters of the LED model are sufficient so it accurately represents the physical LED.



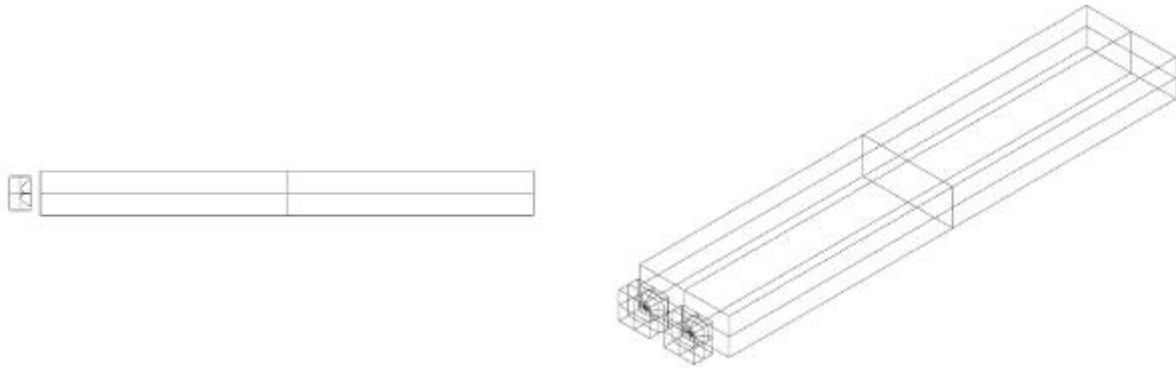
**Figure 6.** Intensity distribution of the LED model. The distribution falls to half of its peak value at 60°.

## 4. LIGHT PIPE SYSTEM MODELING

### 4.1 Direct Input Coupling System Modeling

Three separate light pipe systems are modeled for optical simulation. In all three cases the system geometry is identical except for the light pipe's bottom surface shape. Two surface mounted LEDs, as described above, are used for sources. Two LEDs are used in order to provide uniform light filling of the light pipe in the width dimension. They are rotated 90° to simulate right angle packages. The LEDs are evenly spaced and positioned at a distance of 1 mm to the input surface. The light pipe material is polycarbonate with an index of refraction  $n_{\text{lightpipe}}=1.59$ . The light pipes are 10 mm by 50 mm by 4 mm.

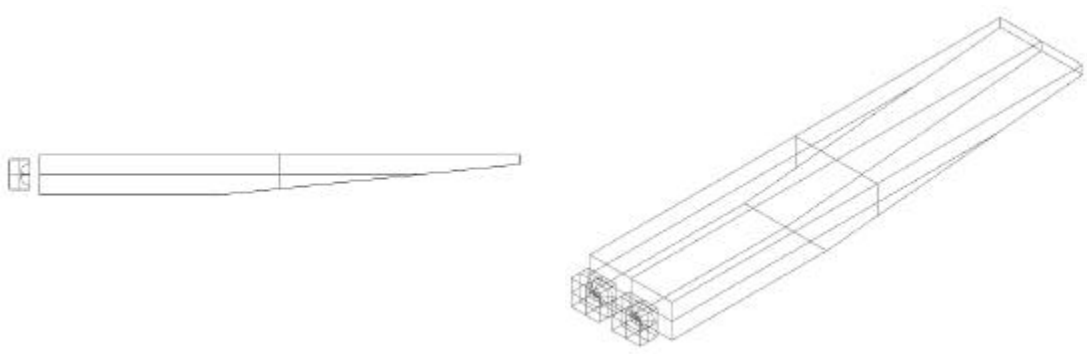
Figure 7 shows a side and an isometric view of the slab or flat bottom surface light pipe system model. The LED source placement and relative dimensions of the system are evident. The bottom surface of the light pipe is flat and remains parallel to the top surface. The bottom surface is assumed to be pad printed with a white reflective paint. This is treated as a scattering surface. The scattering profile used here mimics paint on polished polycarbonate and has been shown to be accurate in other applications such as instrument cluster pointers.[6] All other surfaces of the light pipe are treated as specular. TIR and Fresnel losses are considered in the model.



**Figure 7.** Side and isometric view of the flat bottom surface light pipe system model.

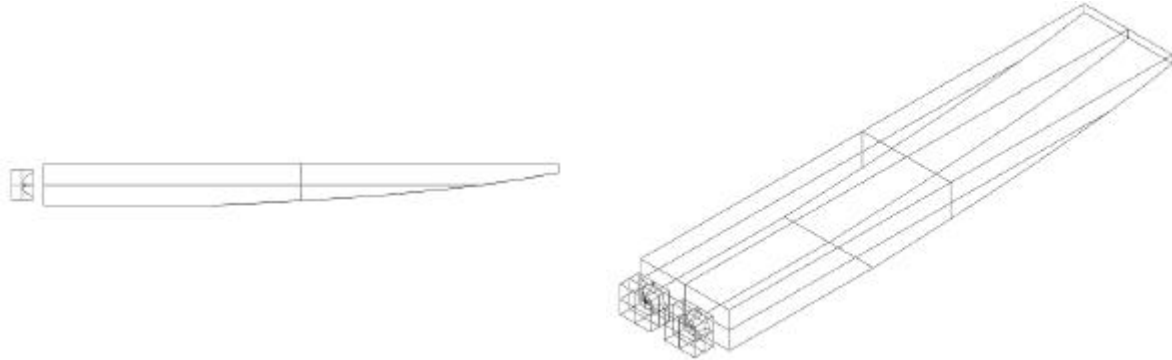
A detector plane is placed directly above the light pipe output surface in order to analyze the output illuminance distribution. The detector has a 19 by 19 bin mesh. One million rays are started from each LED in this analysis. The number of rays collected, combined with the detector binning geometry, results in a finite sampling error of less than 4%.

Figure 8 shows a side and an isometric view of the wedge or flat bottom surface light pipe system model. In this case the bottom surface remains flat but makes an angle with respect to the top surface. The position on the bottom surface where the surface tilt begins and the amount of tilt angle is optimized to produce the most uniform output illuminance distribution. Optimization is accomplished through a partially automated cycle of modification and analysis in the design software. The bottom surface starts to tilt at 19 mm from the input surface and has an angle of 5°.



**Figure 8.** Side and isometric view of the angled bottom surface light pipe system model.

Figure 9 shows a side and an isometric view of the aspheric bottom surface light pipe system model. In this case the bottom surface is both curved and tilted in the long dimension with respect to the top surface. The position where the aspheric surface begins, the degree of tilt angle, and surface curvature is optimized to produce the most uniform output illuminance distribution. Again, optimization is accomplished through a semi-automated cycle of modification and analysis.



**Figure 9.** Side and isometric view of the aspheric bottom surface light pipe system model.

An aspheric surface is defined as a surface that deviates from a spherical shape. The asphere used to create the light pipe's bottom surface are standard, cylindrical, even polynomial aspheres. The surface is cylindrical since it only has curvature in one dimension. The surface curvature can be expressed using the following equation.[7]

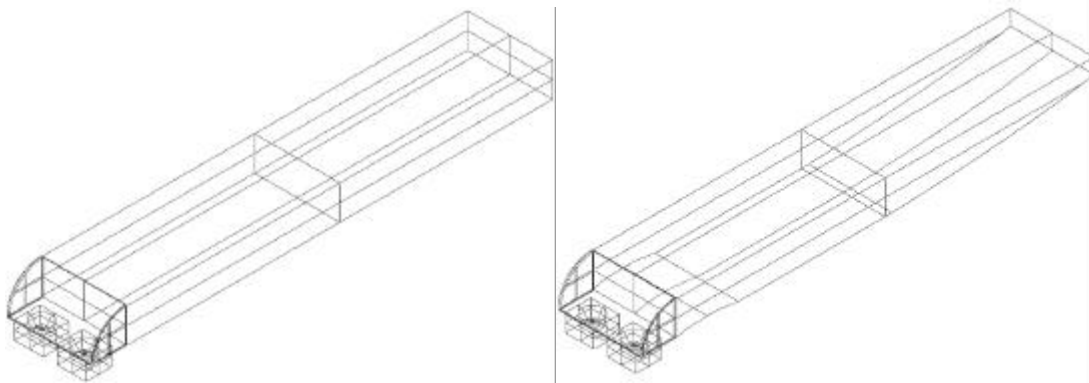
$$z(y) = \frac{cy^2}{1 + \sqrt{1 - (1 - k)c^2y^2}} + Ay^4 + By^6 + Cy^8 \quad (1)$$

In equation (1),  $z$  represents the surface sag and is a function of  $y$ , the height above the optical axis (the axis of symmetry). The curvature (inverse of the radius of curvature) is represented by  $c$ , and the conic constant is represented by  $k$ . Coefficients of the higher order aspheric terms are given by  $A$ ,  $B$ , and  $C$ , and are called the fourth, sixth, and eighth order coefficients, respectively. The surface may consist of higher order terms as specified by the designer. The conic and aspheric terms provide additional degrees of freedom in the design process. The final optimized bottom surface shape consists of a 6<sup>th</sup> order corrected parabolic ( $k=-1$ ) asphere with  $A= 1.7 \times 10^{-8}$  and  $B=5.2 \times 10^{-11}$ . The aspheric surface is tilted  $0.6^\circ$  and intercepts the flat bottom surface at 17.5 mm from the input surface.

#### 4.2 Indirect Input Coupling System Modeling

For the indirect input coupling case the basic system modeling is almost identical to that discussed in section 4.1 for the direct coupling case. However, the LEDs are rotated by  $90^\circ$  and a diffuse reflector is added. Figure 10 shows the isometric view modeled geometry for the slab and aspheric light pipe case. Note that the wedge light pipe geometry is not considered.





**Figure 10.** Isometric views of the indirect input coupling light pipe systems. The LEDs are rotated by 90° and a diffuse reflector is added.

In the slab light pipe case, which acts as a control system, the light pipe geometry is the same as for the direct input coupling. However, after optimization the aspheric bottom surface shape has changed. The final optimized bottom surface shape again consists of a 6<sup>th</sup> order corrected parabolic ( $k=-1$ ) asphere. However, now  $A=3.8 \times 10^{-8}$  and  $B=1.7 \times 10^{-12}$ , the surface is tilted 0.3° and intercepts the flat bottom surface at 15.8 mm from the input surface. Also, due to the input configuration a spherical boss is added to increase uniformity close to the input surface. The boss is placed 8 mm from the input surface, has a curvature of  $0.066 \text{ mm}^{-1}$  and a depth into the light pipe of 0.2 mm.

## 5. SIMULATION RESULTS

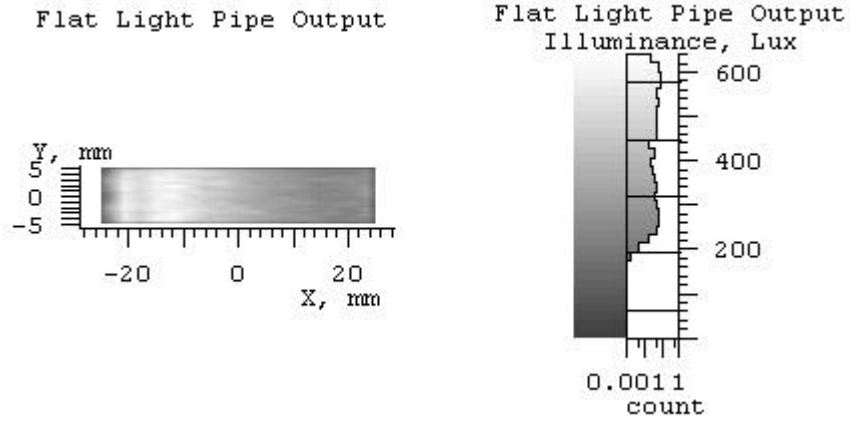
### 5.1 Direct Input Coupling Results

Figures 11 - 13 show the output illuminance distribution for the flat bottom surface, wedge, and aspheric light pipe systems, respectively. The x and y axis correspond to the physical dimensions of the output surface. The output surface illuminance is given in lux by the grayscale values. The scale for the grayscale is given to the right with a histogram showing the number of each value bin. Table 1 gives the total light output, the maximum and average output illuminances, and the output surface illuminance ratios between the input end and middle, and the input and far ends.

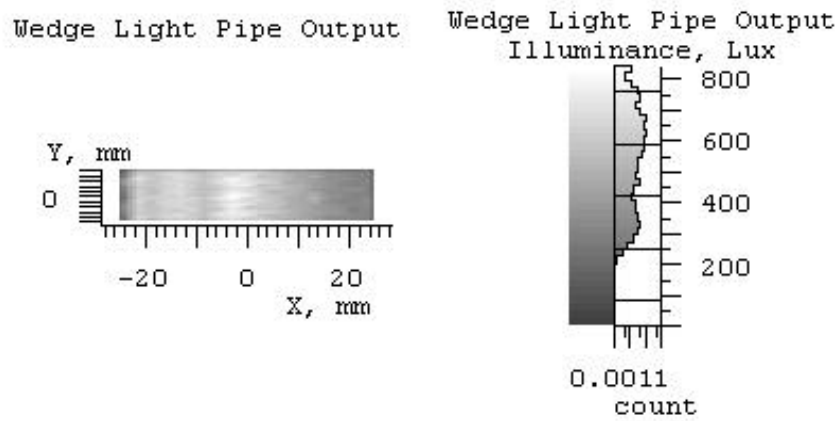
The differences in output uniformity between the geometries are quite evident. In all cases there is a small area near the inputs surface where there is little light output. In this region the input distribution from the LED has little time to scatter and output couple. This is a physical limitation of these types of systems and would be present to some extent in most designs. In practical applications this region would be masked off. Therefore, this region is not considered here when evaluating uniformity.

For the flat bottom surface system the output illuminance varies considerably. The illuminance is high at the input end and decrease continuously across the output surface. At the far end the illuminance is reduced by a factor of three. This degree of uniformity is unacceptable for display applications. As is often done in practice, to increase uniformity two more LEDs could be added to the system. However, this would add cost and complexity to the system.

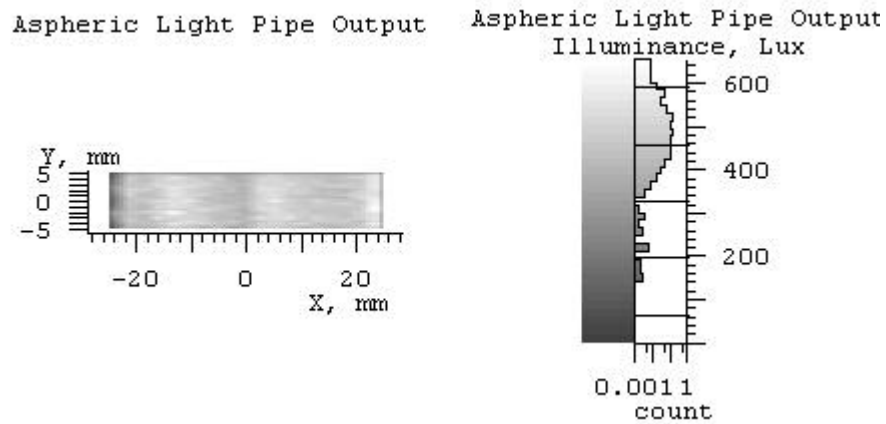
For the wedge light pipe system there is an improvement in optical performance over the slab light pipe case. The total amount of output light has increased by 35%, from 0.20 lm to 0.27 lm. The average illuminance has also increased by 35%, from 395 lx to 530 lx. The light is output coupled from the light pipe more efficiently. The uniformity of the output illuminance distribution has also increased. The end to end illuminance ratio has reduced from 3:1 to 2.6:1. While this is an improvement in uniformity it is still unacceptable. In addition, with the wedge geometry it is not possible to add additional LEDs to increase uniformity.



**Figure 11.** Output illuminance distribution for the flat bottom surface light pipe system.



**Figure 12.** Output illuminance distribution for the wedge light pipe system.



**Figure 13.** Output illuminance distribution for the aspheric light pipe system.

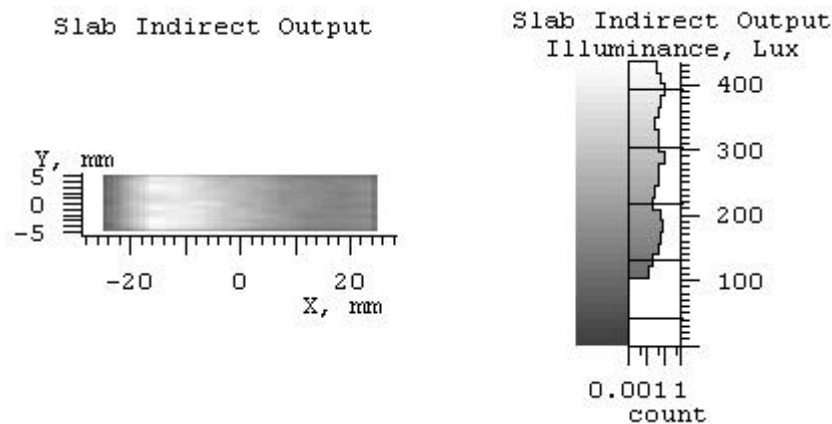
Light Pipe Geometry	Total Light Output (lm)	Max. Illuminance (lux)	Avg. Illuminance (lux)	Input to Middle Ratio	Input to Far End Ratio
Flat	0.20	640	395	1.5:1	3:1
Wedge	0.27	840	530	1.2:1	2.6:1
Aspheric	0.27	770	531	1.1:1	1:1.3

**Table 1.** LED illuminated light pipe system output comparison– direct input coupling.

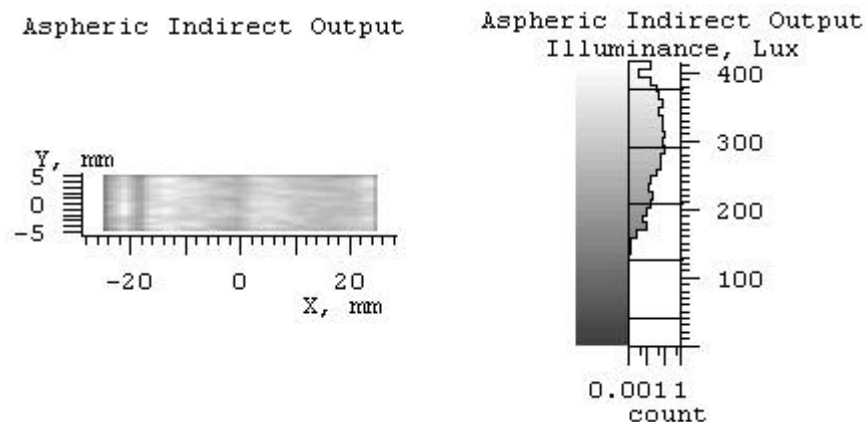
The aspheric light pipe system results in superior optical performance with greater total light output and uniformity than the other cases. The total light output and average output illuminance is the approximately the same as the wedge case, 0.27 lm and 531 lx respectively. This design is also efficiently output coupling the light. However, as can be seen in Figure 13, now the output illuminance distribution uniformity has increased dramatically. The input to middle illuminance ratio is reduced and the input to far end ratio has decreased dramatically, from 3:1 in the slab case to 1:1.3. This degree of uniformity is acceptable for LCD backlighting and would need little or no further diffusion.

## 5.2 Indirect Input Coupling Results

Figures 14 - 15 show the output illuminance distribution for the flat bottom surface and aspheric light pipe systems, respectively. Table 2 gives; the total light output, the maximum and average output illuminance, and the output surface illuminance ratios between; the input end and middle, and the input and far ends.



**Figure 14.** Output illuminance distribution for the slab light pipe system



**Figure 15.** Output illuminance distribution for the aspheric light pipe system

Light Pipe Geometry	Total Light Output (lm)	Max. Illuminance (lux)	Avg. Illuminance (lux)	Input to Middle Ratio	Input to Far End Ratio
Flat	0.13	435	251	1.2:1	2.5:1
Aspheric	0.15	417	297	1.1:1	1:1

**Table 2.** LED illuminated light pipe output comparison – indirect input coupling.

For both output distributions there is a relatively large “dead” area near the input surface. This is a result of the input coupling geometry and this area was not considered when evaluating illuminance uniformity. To overcome this limitation, a second surface modification was added to the system. A spherical boss was used to help output couple more light in this region. This modification was included to show its practical application and to make the example system have validity.

As in the direct input coupling case, the aspheric light pipe has improved optical performance over flat bottomed light pipe. Indirect coupling is inherently less efficient than direct input coupling. There is a reduction in total output of ~45% between the two input geometries. However, the aspheric geometry is still more efficient at output coupling the light. The aspheric light pipe has a total light output and average illuminance that is ~18% greater than the slab light pipe.

With indirect coupling the light incident to the light pipe is already considerably more uniform. This increases the achievable output uniformities. This is evident from Figures 14 and 15 and in the values in Table 2. Again, however, the aspheric case performs better. The uniformity for the slab light pipe is improved overall, but would still not be acceptable. The uniformity for the aspheric light pipe is very good, with an end to end illuminance ratio of ~1:1. This output would be very acceptable for LCD backlighting and would need very little further diffusion.

## 6. CONCLUSIONS

LED light pipe systems for backlighting LCDs were explored through computer modeling. It was shown that systems using aspherical output coupling surface provide better optical performance in both total light output and output illuminance uniformity than linear or angled systems. This was shown for two LED input coupling geometries, direct input coupling and indirect coupling through a diffuse reflector. Two input coupling geometries were used to illustrate the validity of this approach for different input coupling scenarios, and by extrapolation, for different LED light output distributions. For both input coupling geometries, the aspheric light pipe systems converted the input light distribution from the LED sources to a uniform output illuminance distribution over the display area. The output distributions from these systems were acceptable for LCD applications with little or no further conditioning, reducing system costs and complexity.

For the sake of discussion, this analysis considered only a limited set of system geometries. However, these results are generic and can be applied to geometries other than discussed here. For example, a 100 mm long display area might be uniformly illuminated by using two aspheric surface light pipes back to back. For wider display areas a toroidal aspheric bottom surface, with different curvatures in each dimension, may be employed instead of a cylindrical aspheric surface.

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